Theory of the Quadrant Electrometer

The discussion, below, closely follows that given on You Tube by Dr R. Sugaraj Samuel, Electricity and Basic Electronics Part – 12. Reference is made to Figure 1 in this presentation.

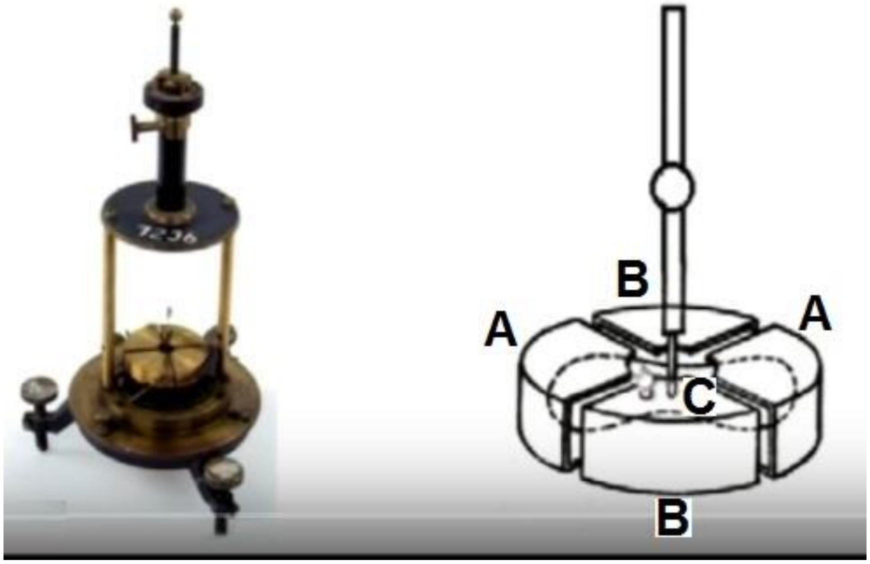

Figure 1 A Quadrant Electrometer showing the electrode configuration

The instrument consists of four electrodes, A, B, A, B as quadrants and a suspended wafer, electrode C, arranged as in Fig 1. Figure 2 shows a simplified diagram. Whereas Figure 1 shows the wafer electrode encased within upper and lower electrodes, the following theory considers only the lower four quadrants as in Figure 2.

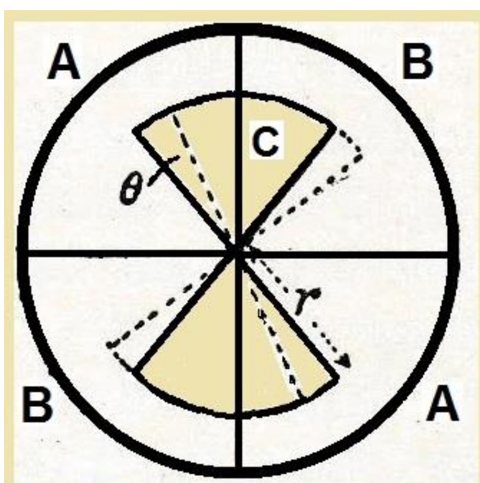

Fig 2A Plan view of the electrode arrangement of a quadrant electrometer

Let the two A-electrodes be connected together and similarly for the B-electrodes. Suppose a high potential, V_C , be applied to the wafer electrode, C, and the pair of quadrants AA and BB be connected to potentials V_A and V_B . The wafer will move from its equilibrium position (shaded) to a new position (dotted line) though an angle θ radians if there is a difference between V_A and V_B.

The analysis (with fringe fields neglected) is as follows:-

The area CHANGE due to the wafer movement is $\frac{1}{2}r^2\theta$ for each arm so that BB gains a covering area of $r^2\theta$ and AA looses the same amount. This causes a change in the capacitance, ΔC, where

$$\Delta C = \varepsilon_0 r^2\theta / d$$

d is the separation of the electrodes.

Now the change of energy ΔU is given by ½(ΔC)V^2 .

The increase in energy in the BB-C capacitor is

$$\Delta U_B = \frac{\varepsilon_0 r^2\theta}{2d}(V_C - V_B)^2$$

the decrease in energy of the AA-C capacitor is

$$\Delta U_A = \frac{\varepsilon_0 r^2\theta}{2d}(V_C - V_A)^2$$

The wafer movement has also twisted the suspension fibre and the energy stored in the fibre is ½ c θ^2 where the fibre has a torsion constant , c .

Therefore we may evaluate the total energy gain as

$$\tfrac{1}{2}c\theta^2 + \frac{\varepsilon_0 r^2\theta}{2d}(V_C - V_B)^2 - \frac{\varepsilon_0 r^2\theta}{2d}(V_C - V_A)^2$$

As the capacitance of the BB-C capacitor increases and the AA-C capacitor decreases, charges are drawn from the battery/ power source to BB-C and restored from AA-C to the battery.

Change of energy for BB-C (charge transfer from electrodes BB to C) is ΔqV or (ΔCV)V i.e. ΔCV^2 . An identical expression is used for AA-C but, of course, the voltage value will be different.

The net energy drawn from the battery is

$$\frac{\varepsilon_0 r^2\theta}{d}(V_C - V_B)^2 - \frac{\varepsilon_0 r^2\theta}{d}(V_C - V_A)^2$$

Now, we can equate the two energies for the equilibrium position:

$$\tfrac{1}{2}c\theta^2 + \frac{\varepsilon_0 r^2\theta}{2d}(V_C - V_B)^2 - \frac{\varepsilon_0 r^2\theta}{2d}(V_C - V_A)^2$$

$$= \frac{\varepsilon_0 r^2\theta}{d}(V_C - V_B)^2 - \frac{\varepsilon_0 r^2\theta}{d}(V_C - V_A)^2$$

Simplification gives

$$\theta = \frac{2\varepsilon_0 r^2}{cd}(V_A - V_B)\left(V_C - \frac{V_A + V_B}{2}\right)$$

We can specify two cases:

(a) if V_C has a large value then we see that θ is proportional to the potential difference between A and B.

(b) we can let V_C and V_A have the same value, namely, V_A . Now the expression becomes

$$\theta = \frac{2\varepsilon_0 r^2}{cd}(V_A - V_B)\left(V_A - \frac{V_A + V_B}{2}\right)$$

so that θ is proportional to the voltage. $(V_A - V_B)^2$.

In summary, the theory predicts that the quadrant electrometer can be quite a versatile instrument.